DO THE $\Delta\phi$ CURVES IN ΔT MEASUREMENT INTERSECT AT A COMMON POINT? LU-220

Hua-shun Zhang

30 April 1993

As pointed [1] that the $\Delta\varphi$ curves in Δt measurement intersect at a common point and this point occurs when $\Delta\varphi_b = \Delta\varphi_a$ and $\Delta W_b = -\Delta W_a$. Now we will discuss what is the condition of this conclusion.

Following the results and signs given in Ref.[2], we have:

$$\begin{pmatrix} \Delta\varphi_b \\ \Delta W_b \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} \Delta\varphi_a \\ \Delta W_a \end{pmatrix} \tag{1}$$

$$\begin{pmatrix} \Delta t_b \\ \Delta t_c \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \begin{pmatrix} \Delta\varphi_a \\ \Delta W_a \end{pmatrix} \tag{2}$$

$$t_{11} = \frac{1-m_{11}}{\omega} \quad \text{(assuming } D_1 = 0\text{)}$$

$$t_{12} = -\frac{m_{12}}{\omega} - \frac{D_{ab}}{E_r c\eta_a^3} = -\frac{m_{12}}{\omega} - \tau_{ab}$$

$$t_{21} = \frac{1-m_{11}}{\omega} + \frac{D_2 m_{21}}{E_r c\eta_b^3} = \frac{1-m_{11}}{\omega} + \tau_{bc}m_{21}$$

$$t_{22} = -\frac{m_{12}}{\omega} - \frac{D_{ab}}{E_r c\eta_a^3} - \frac{D_2}{E_r c}\left(\frac{1}{\eta_a^3} - \frac{m_{22}}{\eta_b^3}\right)$$

Thus

$$\Delta t_b = -\tau_{ab}\Delta W_a + \frac{1-m_{11}}{\omega}\left(\Delta\varphi_a - \frac{m_{12}}{1-m_{11}}\Delta W_a\right) \tag{3}$$

$$\begin{aligned} \Delta t_c = & \left(\frac{1-m_{11}}{\omega} + \tau_{bc}m_{21}\right)\Delta\varphi_a \\ & - \left(\frac{m_{12}}{\omega} + \tau_{ab} + \tau_{bc}\frac{\eta_b^3}{\eta_a^3} - \tau_{bc}m_{22}\right)\Delta W_a \end{aligned} \tag{4}$$

As known: $m_{11}m_{22} - m_{12}m_{21} = 1$, then

$$\begin{aligned}\Delta t_c &= -\left[\tau_{ab} + \tau_{bc}\left(1 + \frac{\eta_b^3}{\eta_a^3}\right)\right]\Delta W_a + \left(\frac{1-m_{11}}{\omega}\right)\left(\Delta\varphi_a - \frac{m_{12}}{1-m_{11}}\Delta W_a\right) \\ &\quad + m_{21}\tau_{bc}\left[\Delta\varphi_a - \frac{m_{12}(1+m_{22})}{1-m_{11}m_{22}}\Delta W_a\right]\end{aligned} \tag{5}$$

As seen, when $m_{11} = m_{22}$ (Note: this is not the actual situation.),

$$\begin{aligned}\Delta t_c &= -\left[\tau_{ab} + \tau_{bc}\left(1 + \frac{\eta_b^3}{\eta_c^3}\right)\right]\Delta W_a \\ &\quad + \left[\left(\frac{1-m_{11}}{\omega}\right) + m_{21}\tau_{bc}\right]\left[\Delta\varphi_a - \frac{m_{12}}{1-m_{11}}\Delta W_a\right]\end{aligned} \tag{6}$$

From the Eqs. (3) and (6), we know that for a given ΔW_a, at condition of $m_{11} = m_{22}$ and $\Delta\varphi_a = \frac{m_{12}}{1-m_{11}}\Delta W_a$, all the $\Delta\phi$ curves intersect at a common point, independent of the values of m_{ij}, as well as the electric field. And

$$\Delta t_b = -\tau_{ab}\Delta W_a = -\Delta t_b^* \tag{7}$$

$$\Delta t_c = -\left[\tau_{ab} + \tau_{bc}\left(1 + \frac{\eta_b^3}{\eta_a^3}\right)\right]\Delta W_a = -\Delta t_c^* \tag{8}$$

In addition, the physical meaning of this point is:

$$\Delta\varphi_b = m_{11}\Delta\varphi_a + m_{12}\Delta W_a = m_{11}\Delta\varphi_a + \frac{m_{12}(1-m_{11})}{m_{12}}\Delta\varphi_a = \Delta\varphi_a \tag{9}$$

$$\Delta W_b = m_{21}\Delta\varphi_a + m_{22}\Delta W_a = \frac{m_{21}m_{12}}{1-m_{11}}\Delta W_a + m_{22}\Delta W_a = -\Delta W_a \tag{10}$$

The slope of the line composed by all the intersect points is given by:

$$\frac{\Delta t_c}{\Delta t_b} = 1 + \frac{\tau_{bc}}{\tau_{ab}}\left(\frac{\eta_b^3}{\eta_a^3} + 1\right) = 1 + \frac{\tau_{bc}}{\tau_{ab}} + \frac{D_2}{D_{ab}} \tag{11}$$

This slope may give us some information about the motion of the real beam center.

However, actually, $m_{11} \neq m_{22}$, we have:

$$\Delta t_b = -\Delta t_b^* + \frac{1-m_{11}}{\omega}\left(\Delta\varphi_a - \frac{m_{12}}{1-m_{11}}\Delta W_a\right) \tag{12}$$

$$\begin{aligned}\Delta t_c &= -\Delta t_c^* + \frac{1-m_{11}}{\omega}\left(\Delta\varphi_a - \frac{m_{12}}{1-m_{11}}\Delta W_a\right) \\ &\quad + m_{21}\tau_{bc}\left[\Delta\varphi_a - \frac{m_{12}(1+m_{22})}{1-m_{11}m_{22}}\Delta W_a\right]\end{aligned} \tag{13}$$

As shown in Tab.1 and Tab.2, the difference between m_{11} and m_{22} is not negligible. Thus a common intersect point is questionable.

In order to estimate this difference in order of magnitude, using the data for LANL in Ref.[2], Tab.1 gives the results of some modules of LANL, assuming $\Delta W_a/W_a \sim 0.3\%$.

Tab. 1 The difference order for LANL

Module	$A = \frac{m_{12}}{1-m_{11}}$	$B = \frac{m_{12}(1+m_{22})}{1-m_{11}m_{22}}$	$\Delta\varphi_{a_1} - \Delta\varphi_{a_2} = (A-B)\Delta W_a$	$\delta\Delta t_c = m_{21}\tau_{bc}(\Delta\varphi_{a_1} - \Delta\varphi_{a_2})$
5	0.2982	0.2662	0.55°	5.0 ps
12	-0.0376	0.0257	2.14°	7.2 ps
21	-0.1682	-0.1372	1.78°	31.6 ps
25	-0.1821	-0.1735	0.58°	9.2 ps
34	-0.2058	-0.1824	2.2°	16.8 ps
42	-0.1893	-0.1736	1.89°	12.2 ps
45	-0.2005	-0.1902	1.33°	10.4 ps

Using the data for our upgrade tanks[3], the results are shown in Tab.2.

Tab. 2 The difference order for FNL

Module	$A = \frac{m_{12}}{1-m_{11}}$	$B = \frac{m_{12}(1+m_{22})}{1-m_{11}m_{22}}$	$\Delta\varphi_{a_1} - \Delta\varphi_{a_2} = (A-B)\Delta W_a$	$\delta\Delta t_c = m_{21}\tau_{bc}(\Delta\varphi_{a_1} - \Delta\varphi_{a_2})$
11	0.02454	-0.04545	1.5°	-5.8 ps
12	-0.003604	0.1871	5.3°	-4.7 ps
13	-0.02117	0.05139	12.5°	4.1 ps
14	-0.03295	-0.009439	1.75°	6.1 ps
15	-0.04073	-0.02840	0.6°	2,8 ps
16	-0.04523	-0.03732	0.44°	2,4 ps
17	-0.04908	-0.04353	0.35°	2.0 ps

$\delta\Delta t_c$ may give the upper errors of the "intersect points", which determine the error of the slop composed by the "intersect points".

REFERENCES

[1] K. R. Crandall, etc. "Proc. of the Sypm. on 1972 linac", p. 122.
[2] K. R. Crandall, LA-6374-MS.
[3] T. Owen, LU-177.